Η διάσταση του μηδενοχώρου είναι 1, τα διανύσματα είναι τρισδιάστατα εξαιτίας των 3 μεταβλητών.

Όταν δεν έχουμε ελεύθερες μεταβλητές, η λύση είναι το 0.

Άσκηση

$$A=\begin{bmatrix}0&1&2&2\\0&3&8&7\\0&0&4&2\end{bmatrix}=\begin{bmatrix}0&1&2&2\\0&0&2&1\\0&0&4&2\end{bmatrix}=\begin{bmatrix}0&1&2&2\\0&0&2&1\\0&0&0&0\end{bmatrix}\quad rank(A)=2$$

pivots

free

$$range(A)=\left\{\begin{bmatrix}1\\3\\0\end{bmatrix},\begin{bmatrix}2\\8\\4\end{bmatrix}\right\}$$

$$range(A^T)=\left\{\begin{bmatrix}0\\1\\2\\2\end{bmatrix},\begin{bmatrix}0\\0\\2\\1\end{bmatrix}\right\}$$

$Ax=0$ Μηδενόχωρος

$0x_1+x_2+2x_3+2x_4=0$ (2)

$0x_1+0x_2+2x_3+x_4=0$ (1)

Για $x_4=1$, (1) $\Rightarrow x_3=-1/2$

(2) $\Rightarrow x_2=-1$

Για $x_1=1$, (1) $\Rightarrow x_3=0$

(2) $\Rightarrow x_2=0$

Άρα $N(A)=\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\-1\\-1/2\\1\end{bmatrix}\right\}$

Άσκηση

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & -2 & 4 & 0 \\ 3 & -3 & 7 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & -3 & 7 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

pivots free

$$\text{range}(A) = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right\}$$

$$\text{range}(A^T) = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$Ax = 0$

$x_1 - x_2 + 2x_3 + 0x_4 = 0 \quad (2)$

$0x_1 + 0x_2 + x_3 + 0x_4 = 0 \quad (1)$

Για $x_2 = 1$, $(1) \Rightarrow x_3 = 0$

$(2) \Rightarrow x_1 = 1$

Για $x_4 = 1$, $(1) \Rightarrow x_3 = 0$

$(2) \Rightarrow x_1 = 0$

Άρα $N(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

Υπόχωροι του R^3

α) Όλα τα διανύσματα $(x,y,z)^T$, όπου $10x+y+2017z=0$
Σωστό -> Null [10 1 2017]

β) $(x,y,z)^T$, όπου $x+y+z \leq 2017$
Λάθος $c\begin{bmatrix}1\\0\\0\end{bmatrix} \to 2017\begin{bmatrix}1\\0\\0\end{bmatrix}$ δεν ανήκει

γ) $(x,y,z)^T$, όπου $x+y+z=0$ ΚΑΙ $x+2y+3z=0$
Σωστό Null $\begin{bmatrix}1&1&1\\1&2&3\end{bmatrix}$

δ) $(x,y,z)^T$, όπου $x+y+z=0$ Ή $x+2y+3z=0$
Λάθος $\begin{bmatrix}-1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\end{bmatrix} \to \begin{bmatrix}-1\\0\\1\end{bmatrix}+\begin{bmatrix}1\\1\\1\end{bmatrix}$ δεν ανήκει

ε) $(b_1,b_2,b_3)^T$ όπου $\begin{bmatrix}1&1&1\\1&2&3\\3&2&1\end{bmatrix}$, το $x=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ να έχει λύση

29-3-2017

ΚΕΦΑΛΑΙΟ 3.4

Άσκηση 1

$$A=\begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \qquad b=\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 2 & 4 & 6 & 4 & 4 \\ 2 & 5 & 7 & 6 & 3 \\ 2 & 3 & 5 & 2 & 5 \end{array}\right] = \left[\begin{array}{cccc|c} 2 & 4 & 6 & 4 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & -1 & -1 & -2 & 1 \end{array}\right] = \left[\begin{array}{cccc|c} 2 & 4 & 6 & 4 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

rank(A)=2

$$\text{range}(A)=\left\{\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}\right\} \qquad \text{range}(A^T)=\left\{\begin{bmatrix} 2 \\ 4 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}\right\}$$

Φέρνω το μητρώο σε ΑΓΚΜ

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 2 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Έχουμε φέρει το μητρώο σε μια μορφή $A \to R = [I \mid F]$

Για μηδενόχωρο τότε $x = \begin{bmatrix} -F \\ I \end{bmatrix}$ Άρα $N(A) = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

Διαφορετικά:

$x_1 + x_3 - 2x_4 = 0$ (1)

$x_2 + x_3 + 2x_4 = 0$ (2)

Για $x_3 = 1$, (1) $\Rightarrow x_1 = -1$

(2) $\Rightarrow x_2 = -1$

Για $x_4 = 1$, (1) $\Rightarrow x_1 = 2$

(2) $\Rightarrow x_2 = -2$

Άρα $N(A) = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

$Ax = b \Rightarrow x = x_p + c x_s$

Το x_p προκύπτει από την λύση του $Ax = b$, μηδενίζοντας τις ελεύθερες μεταβλητές.

Το x_s προκύπτει από την λύση του $Ax = 0$.

Για x_p έχουμε:

$x_1 + x_3 - 2x_4 = 4$ (3)

$x_2 + x_3 + 2x_4 = -1$ (4)

Για $x_3 = x_4 = 0$, (3) $\Rightarrow x_1 = 4$

(4) $\Rightarrow x_2 = -1$

Άρα $x = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

Χρησιμοποιώ x_3, x_4 για να θυμάμαι από που προέρχονται.

Παραγοντοποίηση τάξης

$A \in mxn$ και $rank(A) = r = 2$

$A = xy$, όπου $x \in mxr$ και $y \in rxn$

Άρα $A = \begin{bmatrix} 2 & 4 \\ 2 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$

Άσκηση 4

$A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 2 & 6 & 4 & 8 & 3 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right] = \left[\begin{array}{ccccc} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$rank(A) = 2 \quad range(A) = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \right\} \quad range(A^T) = \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

$Ax=0$

Για να φέρουμε το μητρώο A στη μορφή $A \to [I|F]$, εναλλάσουμε το x_2 με το x_3.

Άρα $A = \begin{array}{c} \begin{matrix} x_1 & x_3 & x_2 & x_4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$

$x = \begin{bmatrix} -F \\ I \end{bmatrix}$ Άρα $\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ και $\begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

Εναλλάσουμε όμως τα x_2, x_3. Άρα, τελικά $N(A) = \left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

Και $x_s = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + x_4 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

$Ax=b$

$\left.\begin{array}{l} x_1 + 3x_2 = 1/2 \\ x_3 + 2x_4 = 1/2 \end{array}\right\}$ $x_2 = x_4 = 0 \Rightarrow x_1 = 1/2$

$x_3 = 1/2$

Άρα $x = x_p + x_s = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Άσκηση 5

$$\begin{aligned} x+2y-2z &= b_1 \\ 2x+5y-4z &= b_2 \\ 4x+9y-8z &= b_3 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & 2 & -2 & b_1 \\ 2 & 5 & -4 & b_2 \\ 4 & 9 & -8 & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & -2 & b_1 \\ 0 & 1 & 0 & b_2-2b_1 \\ 0 & 1 & 0 & b_3-4b_1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -2 & b_1 \\ 0 & 1 & 0 & b_2-2b_1 \\ 0 & 0 & 0 & b_3-b_2-2b_1 \end{array}\right]$$

Πρέπει $b_3-b_2-2b_1=0$

$$= \begin{bmatrix} 1 & 0 & -2 & 5b_1-2b_2 \\ 0 & 1 & 0 & b_2-2b_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$Ax=0$

$N(A)=x_s=\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} x_3$

$Ax=b$

$x_1-2x_3=5b_1-2b_2 \overset{x_3=0}{\Longrightarrow} x_1=5b_1-2b_2$

$x_2=b_2-2b_1$

Άρα $x=x_p+x_s=\begin{bmatrix} 5b_1-2b_2 \\ b_2-2b_1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

Άσκηση

Κατασκευή 3x3 μητρώου με $N(A)=\begin{bmatrix}1\\2\\1\end{bmatrix}$ και $range(A)=\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\1\end{bmatrix}\right\}$

Άρα $A=\begin{bmatrix}1&1&b_1\\1&2&b_2\\1&1&b_3\end{bmatrix}=\begin{bmatrix}1&1&b_1\\0&1&b_2-b_1\\0&0&b_3-b_1\end{bmatrix}$ Πρέπει $b_3-b_1=0 \Rightarrow b_3=b_1$

$=\begin{bmatrix}1&0&2b_1-b_2\\0&1&b_2-b_1\\0&0&0\end{bmatrix}$ Άρα $x_s=N(A)=\begin{bmatrix}b_2-2b_1\\b_1-b_2\\1\end{bmatrix}=\begin{bmatrix}1\\2\\1\end{bmatrix} \Rightarrow \begin{matrix}b_2-2b_1=1\\b_1-b_2=2\end{matrix}$

Άρα $b_2=-5$ και $b_1=b_3=-3$

Τελικά $A=\begin{bmatrix}1&1&-3\\1&2&-5\\1&1&-3\end{bmatrix}$

ΚΕΦΑΛΑΙΟ 4.1

Άσκηση 10

$P=\frac{aa^T}{a^Ta}$ $a=$ διάνυσμα/στήλη $P=A(A^TA)^{-1}A^T$

α) $A^T=A$

$N(A) \perp range(A^T)=range(A)$

β) $Ax=0 \rightarrow N(A)$ $Az=5z \rightarrow range(A)$

ΚΕΦΑΛΑΙΟ 4.2

Άσκηση 1

$b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $P = \frac{b^T a}{a^T a} a = \frac{5}{3} a$

εσωτερικό γινόμενο

$e = P - b = \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix}$ $ae = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix} = 0$

Φροντιστήριο 7-4-2017

ΚΕΦΑΛΑΙΟ 4.2

Άσκηση 11

$A^T A \hat{x} = A^T b$ (1)

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $A^T A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$A^T b = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$$(1) \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 3 \end{array}\right] \rightarrow \begin{matrix} x_1 = -1 \\ x_2 = 3 \end{matrix} \quad \hat{x} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$A \quad x = b \Rightarrow x = A^{-1}b$$

Άλλος τρόπος

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$p = A\hat{x} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \qquad e = b - p = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

Άσκηση 12

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix} \qquad A^TA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 8 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}\hat{x} = \begin{bmatrix} 8 \\ 14 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 2 & 2 & 8 \\ 0 & 1 & 6 \end{array}\right] \rightarrow \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

$$P = A\hat{x} \Rightarrow P = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix} \rightarrow e = b - p = 0$$

$$P = A(A^TA)^{-1}A^T \rightarrow p = Pb \rightarrow$$

a) $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $P_2 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Ιδιοτητες Οριζουσας $\det(A+B) \neq \det(A)+\det(B)$

$\det(6A) = 6^n \det(A)$

Αn.xn

$\det(AB) = \det(A)\det(B)$

5.1

① $A_{4\times4}$, $\det(A)=|A|=\frac{1}{2}$

$\det(2A)=2^4|A|=\frac{16}{2}=8$ $\det(-A)=(-1)^2\det(+A)=\det(A)=\frac{1}{2}$

$\det(A^2)=\det(AA)=\det(A)\det(A)=\frac{1}{4}$

$\det(A^{-1})=\frac{1}{\det(A)}=2$

③

α) $\det(I+A)=1+|A|$ Λάθος $\left|\begin{bmatrix}1&1\\0&2\end{bmatrix}\right| \to 2$ $\left|\begin{bmatrix}2&1\\0&3\end{bmatrix}\right|=6$

β) $\det(ABC)=|A||B||C|$ Σωστό $|AB||C|=|A||B||C|$

γ) $\det(4A)=4|A|$ Λάθος

δ) $\det(AB-BA)=0$ Λάθος δεν ισχύει η αντιμεταθετική ιδιότητα

(18)

$$\begin{bmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{bmatrix}=(b-a)(c-a)(c-b)$$

→ 1) Τριγωνικό μητρώο
→ Γινόμενο διαγων...
2) Μεγάλος τύπος
3) Αλγεβρικά συμπληρώματ...

$$\begin{bmatrix}1&a&a^2\\0&b-a&b^2-a^2\\0&c-a&c^2-a^2\end{bmatrix}\to\begin{bmatrix}1&a&a^2\\0&b-a&b-a^2\\0&0&(c^2-a^2)-\frac{(b^2-a^2)(c-a)}{b-a}\end{bmatrix}$$

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$$|A| = (b-a)\left((c^2-a^2) - \frac{(b^2-a^2)(c-a)}{(b-a)}\right) = (b-a)\left((c-a)(c+a) - \frac{\cancel{(b-a)}(b+a)(c-a)}{\cancel{b-a}}\right)$$

$$= (b-a)(c-a)(c+a-b-a) = (b-a)(c-a)(c-b$$

5.2. (1)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} = (-1)^{1+1}\cdot 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + (-1)^{1+2}\, 2 \begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix}$$

$$+ (-1)^{1+3}\, 3 \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} = (1-4) - 2(3-6) + 3(6-3) = -3+6+9 = 12$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix} = (-1)^{1+1}\cdot 1 \begin{vmatrix} 4 & 4 \\ 6 & 7 \end{vmatrix} + (-1)^{1+2}\, 2 \begin{vmatrix} 4 & 4 \\ 5 & 7 \end{vmatrix} + (-1)^{1+3}\, 3 \begin{vmatrix} 4 & 4 \\ 5 & 6 \end{vmatrix} = 0$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

3) $A = \begin{bmatrix} x & x & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix} \to$ diag $= 0$

$$x \begin{vmatrix} 0 & x \\ 0 & x \end{vmatrix} - 0 \begin{vmatrix} & \\ & \end{vmatrix} + 0 \begin{vmatrix} & \\ & \end{vmatrix} = 0$$

απαλοιφή $\begin{bmatrix} x & x & x \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}$ rank(A) = 2 όχι αντιστρέψιμο

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(25) 2x2 μπλοκ 4x4 μητρώο $\left.\right\}$ $\begin{bmatrix} A & B \\ 0 & D \end{bmatrix} \to \begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A||D|$ Σωστό

Το αντιμετωπίζουμε ως τριγωνικό μητρώο, άρα det $= |A||D|$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq \overset{|A||D| - |C||B|}{\cancel{|A||B| - |C||D|}}$$

αντιπαράδειγμα $\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 0 \end{vmatrix}$, $|A|=2$, $|B|=1$ $|C|=1$ $|D|=-6$

$|A||B|-|C||D| = \cancel{8}\ -5$

$$\to 1 \begin{vmatrix} 2 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 3 & 0 \end{vmatrix} - 0 \cancel{\begin{vmatrix} \; \end{vmatrix}} + 1 \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 0 \end{vmatrix} - 0 \cancel{\begin{vmatrix} \; \end{vmatrix}} =$$

$$= 2 \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} + \cancel{\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}} - \begin{vmatrix} 2 & 2 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -12 - 2 + 5 = -9$$

det (AD - CB)

$$\left.\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} &= \begin{bmatrix} 1 & 2 \\ 6 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \end{aligned}\right\} \begin{bmatrix} 1 & 2 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & -2 \end{bmatrix} =$$

-6

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(30) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$ $\left.\begin{matrix} \\ \\ - \quad - \; - \; + \; + \; + \end{matrix}\right\}$ $|A| = 159 + 267 + 348 - 357 - 168 - 249$
$= 0$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$ γραμμικά εξαρτημένη
$|A| = 0$

cramer $Ax = b$

$\left.\begin{matrix} 2x_1 + 5x_2 = 1 \\ x_1 + 4x_2 = 2 \end{matrix}\right\}$ $A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $Ax = b$

$|A| = 3$

$B_1 = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$ $B_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$|B_1| = -6$ $|B_2| = 3$

$\left.\begin{matrix} x_1 = \dfrac{|B_1|}{|A|} = \dfrac{-2}{1} \\ \\ x_2 = \dfrac{|B_2|}{|A|} = 1 \end{matrix}\right\}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ +1 \end{bmatrix}$

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$$\left.\begin{aligned} 2x_1 + x_2 \quad &= 1 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0 \end{aligned}\right\} \quad A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|A| = 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 4$$

$$B_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \qquad |B_1| = 3$$

$$B_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad |B_2| = -2$$

$$B_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad |B_3| = 1$$

$$x_1 = \frac{|B_1|}{|A|} = \frac{3}{4} \qquad x_2 = \frac{|B_2|}{|A|} = -\frac{1}{2} \qquad x_3 = \frac{|B_3|}{|A|} = \frac{1}{4}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3/4 \\ -1/2 \\ 1/4 \end{bmatrix}$$

$$Ax=b = x_1a_1 + x_2a_2 + x_3a_3$$

$$|B_1| = |b \quad a_2 \quad a_3| = |x_1a_1 + x_2a_2 + x_3a_3 \quad a_2 \quad a_3| =$$

$$= x_1 |a_1 \quad a_2 \quad a_3| = x_1 \det(A)$$

NO.

Date

1

Sylvester

$\det(I_m + CD) = \det(I_n + DC)$

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 6 & 10 \end{bmatrix} \Rightarrow I_3 + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} = vv^T \qquad \nearrow v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\det(A) = \det(I_3 + vv^T) = \det(1 + v^T v) = 15$

Εξετάσεις: $\det(I+0)$ ή $\det(I+P)$

6.1 ②

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \to \left| \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix} \right| \to p(\lambda) = (1-\lambda)(3-\lambda) - 8 =$$

$= 3 - 4\lambda + \lambda^2 - 8 = \lambda^2 - 4\lambda - 5 = (\lambda-5)(\lambda+1)$

Για $\lambda_1 = -1 \Rightarrow \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \to \begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x_1 + 4x_2 = 0 \Rightarrow x_1 = -\frac{4}{2} x_2 \Rightarrow x_1 = -2x_2$ $\qquad v_1 \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2$

Για $\lambda_2 = 5 \to \begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -4 & 4 \\ 0 & 0 \end{bmatrix} x = 0$

$-4x_1 + 4x_2 = 0$ $\qquad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x_1 = x_2$

$x_1 = x_2 \quad \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}x_1\\x_1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}x_1$

$A=\begin{bmatrix}2&4\\2&4\end{bmatrix} \to \left|\begin{bmatrix}2-\lambda&4\\2&4-\lambda\end{bmatrix}\right| \to (2-\lambda)(4-\lambda)-8 = -6\lambda+\lambda^2 = \lambda(\lambda-6) \to \lambda_1=0,\ \lambda_2=6$

$\lambda_1=0 \to Ax=0 \to 2x_1+4x_2=0 \to x_1=-2x_2 \Rightarrow x_2=-\frac{1}{2}x_1 \quad v_1=\begin{bmatrix}1\\-\frac{1}{2}\end{bmatrix}x_1$

Με ελευθερη μεταβλητή, βγαίνει $v_1=\begin{bmatrix}-2\\1\end{bmatrix}x_2$

$\lambda_2=6 \to \begin{bmatrix}-4&4\\2&-2\end{bmatrix}x=0 \Rightarrow v_2=\begin{bmatrix}1\\1\end{bmatrix}$

$A=\begin{bmatrix}-1&3\\2&0\end{bmatrix} \Rightarrow \left|\begin{bmatrix}-1-\lambda&3\\2&-\lambda\end{bmatrix}\right| \to p(\lambda)=\lambda^2+\lambda-6 \to \lambda_1=2,\ \lambda_2=-3$

$A(\lambda_1) \to \lambda_1=2 \to v_1=\begin{bmatrix}1\\1\end{bmatrix}$

$A(\lambda_2) \to \lambda_2=-3 \to v_2=\begin{bmatrix}1\\-3/2\end{bmatrix}$

$A^2=\begin{bmatrix}7&-3\\-2&6\end{bmatrix} \to \lambda_1=9,\ \lambda_2=4 \ \Big| \to v_1=\begin{bmatrix}1\\-3/2\end{bmatrix} \quad v_2=\begin{bmatrix}1\\1\end{bmatrix}$

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21 $\qquad a,b,c \to |A-\lambda I| = 9-\lambda^3$

$$A=\begin{bmatrix}0&1&0\\0&0&1\\a&b&c\end{bmatrix} \Rightarrow \begin{bmatrix}-\lambda&1&0\\0&-\lambda&1\\a&b&c-\lambda\end{bmatrix} \to -\lambda\begin{vmatrix}-\lambda&1\\b&c-\lambda\end{vmatrix} + \begin{vmatrix}0&1\\a&c-\lambda\end{vmatrix}$$

$$= -\lambda(-\lambda(c-\lambda)-b)+a = -\lambda(-c\lambda+\lambda^2-b)+a = -\lambda^3+c\lambda^2+\lambda b+a$$
$$= -\lambda^3+9\lambda$$

$c=0,\ b=+9\quad a=0$

$$A=\begin{bmatrix}0&1&0\\0&0&1\\a&b&c\end{bmatrix} \qquad A=\begin{bmatrix}0&1&0\\0&0&1\\0&9&0\end{bmatrix}$$

$$\begin{bmatrix}-\lambda&1&0\\0&-\lambda&1\\0&9&-\lambda\end{bmatrix} \to -\lambda^3+9\lambda = \lambda(9-\lambda^2) \to \lambda_1=0$$
$$\lambda_{2,3}=\pm 3$$

$$v_1=\begin{bmatrix}1\\0\\0\end{bmatrix}\quad v_2=\begin{bmatrix}1\\3\\9\end{bmatrix}\quad v_3=\begin{bmatrix}1\\-3\\9\end{bmatrix}$$

30

$$A = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 6 \\ 0 & 0 & 6-\lambda \end{bmatrix} \quad p(\lambda) = (1-\lambda)(4-\lambda)(6-\lambda) = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = 4 \quad \lambda_3 = 6$$

$$A(\lambda_1) \to \begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 5 \end{bmatrix} x = 0 \Rightarrow v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A(\lambda_2) \to \begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix} x = 0 \Rightarrow \begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} x = 0 \quad \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix}$$

$$A(\lambda_3) = \begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & 0 \end{bmatrix} x = 0 \to \begin{bmatrix} 9/5 \\ 3 \\ 1 \end{bmatrix} \times 3$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} -\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 3 & 0 & -\lambda \end{bmatrix} \to p(\lambda) = (\lambda-2)(\lambda^2-3)$$

$$\lambda_1 = 2 \quad \lambda_{2,3} = \pm\sqrt{3}$$

$$B(\lambda_1) = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 0 & 0 \\ 3 & 0 & -2 \end{bmatrix} x = 0 \to v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$B(\lambda_2) = \begin{bmatrix} -\sqrt{3} & 0 & 1 \\ 0 & 2-\sqrt{3} & 0 \\ 3 & 0 & -\sqrt{3} \end{bmatrix} \to v_2 = \begin{bmatrix} \sqrt{3} \\ 0 \\ 1 \end{bmatrix}$$

$$B(\lambda_3) \longrightarrow v_3 = \begin{bmatrix} -\sqrt{3} \\ 0 \\ 1 \end{bmatrix}$$

NO.

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Cayley-Hamilton

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \to p(\lambda) = \lambda^2 - 4\lambda + 3 = 0$$

$$p(A) = 0 \to A^2 - 4A + 3I = 0 \xRightarrow{\cdot A^{-1}} A - 4 + 3A^{-1} = 0 \Rightarrow A^{-1} = -\frac{1}{3}(A - 4I)$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & \overset{5}{(10000)} \end{bmatrix} = \begin{bmatrix} A_1 & & \\ & A_2 & \\ & & A_3 \end{bmatrix}$$

$$\lambda = [-0{,}2361 \mid 1{,}382 \mid 3{,}618 \mid 4{,}2361 \mid 5]$$

$$V = \begin{bmatrix} -0{,}8507 & 0 & 0 & 0{,}52 & 0 \\ 0{,}1527 & 0 & 0 & 0{,}85 & 0 \\ 0 & -0{,}5257 & -0{,}85 & 0 & 0 \\ 0 & -0{,}85 & -0{,}52 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix} \longrightarrow \lambda_1=1 \quad \lambda_2=3$$

$$A(\lambda_1)x=0 \Rightarrow \begin{bmatrix}0 & 2\\ 0 & 2\end{bmatrix}x=0 \Rightarrow v_1=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$A(\lambda_2)x=0 \quad \begin{bmatrix}-2 & 2\\ 0 & 0\end{bmatrix}x=0 \rightarrow v_2=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$A=\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix} \rightarrow \lambda_1=0$$

$$v_2=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$$ κανονικοποιημενο

$$X=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}, \quad \Lambda=\begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}$$

$$X^{-1}=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

$$A=X\Lambda X^{-1}$$
$$\Lambda=X^{-1}AX$$

$$[X]\begin{bmatrix}\lambda_1 & \\ & \lambda_n\end{bmatrix}[X^{-1}]$$

$$A^{5000}=X\Lambda^{5000}X^{-1}=X\begin{bmatrix}1 & 0\\ 0 & 3^{5000}\end{bmatrix}X^{-1}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3} \rightarrow B^T B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 6 \\ 3 & 6 & 9-\lambda \end{bmatrix}$$

$$p(\lambda) = \lambda^2(\lambda - 14) \rightarrow \lambda_{1,2} = 0$$
$$\lambda_3 = 14$$

$$V = \begin{bmatrix} -0{,}89 & 0{,}35 & 0{,}26 \\ 0{,}44 & 0{,}71 & 0{,}53 \\ 0 & -0{,}59 & -0{,}80 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 9 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 3-\lambda & 9 \\ 4 & -3-\lambda \end{bmatrix}$$

$$p(\lambda) = \lambda^2 + 27 \rightarrow \lambda_{1,2} = \pm 3\sqrt{3}i$$

$$A(\lambda_1)\begin{bmatrix} 3-3\sqrt{3}i & 9 \\ 4 & -3-3\sqrt{3}i \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ \frac{1}{3}(1-\sqrt{3})x_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ \frac{1}{3}(1-\sqrt{3}i) \end{bmatrix} x_1$$

$$A(\lambda_2) \rightarrow v_2 = \begin{bmatrix} 3 \\ 1+\sqrt{3}i \end{bmatrix}$$

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$$B = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix} \quad p(\lambda) = \lambda^2 + 4 \quad \lambda_{1,2} = \pm 2i$$

$$\lambda_1 \begin{bmatrix} -2i & -1 \\ 4 & -2 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -2i \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

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$V \to W \quad T(V) = W \quad x, y \in V \to T(x,y) = T(x) + T(y)$

$T(cx) = cT(x)$

$T(0) = 0$

7.1

③ $T(v) \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad v \in V$

$x, y \in V \quad T(cx+dy) = T\left(c\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + d\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} cx_1 + dy_1 \\ cx_2 + dy_2 \end{bmatrix}\right)$

$= (cx_2 + dy_2, cx_1 + dy_1) = (cx_2, cx_1) + (dy_2, dy_1) =$

$= cT(x) + dT(y)$

β) $T(v) = (v_1, v_2)$

$T(cx+dy) = T\begin{pmatrix} cx_1 + dy_1 \\ cx_2 + dy_2 \end{pmatrix} = (cx_1 + dy_1, cx_1 + dy_1) =$

$(cx_1, cx_1) + (dy_1, dy_1) = cT(x) + d(T(y)$

γ) $T(v) = (0, v_1) \quad T(0)$

$T(cx+dy) = T\begin{pmatrix} cx_1 + dy_1 \\ cx_2 + dy_2 \end{pmatrix} = (0, cx_1 + dy_1) = (0, cx_1) +$

$+ (0, dy_1) = cT(x) + dT(y)$

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Date

δ) $T(v) = (0,1)$ $\qquad T(0) \neq 0$

$$T(cx+dy) = T\begin{pmatrix} cx_1+dy_1 \\ cx_2+dy_2 \end{pmatrix} = (0,1) \neq cT(x) + dT(y)$$

⑦ $V = R^2,\ W = R^2$

a) $T(v) = -V$ $\qquad T(T(V))$

$$T\begin{pmatrix} cx_1+dy_1 \\ cx_2+dy_2 \end{pmatrix} = (-(cx_1+dy_1), -(cx_2+dy_2)) =$$

$$= -c(x_1, x_2) - d(y_1, y_2)$$

$$T(T(v)) \Rightarrow T(T(cx+dy)) = T\left(T\begin{pmatrix} cx_1+dy_1 \\ cx_2+dy_2 \end{pmatrix}\right) =$$

$$= T\left(-\begin{pmatrix} cx_1+dy_1 \\ cx_2+dy_2 \end{pmatrix}\right) = (cx_1, cx_2) + d(y_1, y_2) = T(x) + T(y)$$

б) $T(v) = v + (1,1)$ $\qquad T(0) \neq 0$

$$T(T(v)) = T(v+(1,1)) \Rightarrow T\begin{pmatrix} cx_1+dy_1+1 \\ c2+dy_2+1 \end{pmatrix} =$$

$$= \begin{bmatrix} cx_1+dy_1+1 \\ cx_2+dy_2+1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underbrace{c\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{T(x)} + \underbrace{d\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{T(y)}$$

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γ) $T(v) = (-v_2, v_1)$

$$T(T(v)) = T\left(T\begin{pmatrix} cx_1 + dy_1 \\ cx_2 + dy_2 \end{pmatrix}\right) = T\begin{pmatrix} -(cx_2 + dy_2) \\ (cx_1 + dy_1) \end{pmatrix} =$$

$$= \begin{pmatrix} -(cx_1 + dy_1) \\ -(cx_2 + dy_2) \end{pmatrix} = \underset{T(x)}{-(cx} + \underset{T(y)}{dy)}$$

δ) $T(v) = \left(\frac{v_1 + v_2}{2}, \frac{v_1 + v_2}{2}\right)$

$$T\begin{pmatrix} cx_1 + dy_1 \\ cx_2 + dy_2 \end{pmatrix} = \begin{pmatrix} \frac{cx_1 + dy_1 + cx_2 + dy_2}{2} \\ \frac{cx_1 + dy_1 + cx_2 + dy_2}{2} \end{pmatrix} =$$

$$= c\begin{pmatrix} \frac{x_1 + x_2}{2} \\ \frac{x_1 + x_2}{2} \end{pmatrix}, d\begin{pmatrix} \frac{y_1 + y_2}{2} \\ \frac{y_1 + y_2}{2} \end{pmatrix}$$

$$T(T(v) \Rightarrow T\left(T\begin{pmatrix} cx_1 + dy_1 \\ cx_2 + dy_2 \end{pmatrix}\right) = T\begin{pmatrix} \frac{cx_1 + dy_1 + cx_2 + dy_2}{2} \\ \frac{cx_1 + dy_1 + cx_2 + dy_2}{2} \end{pmatrix}$$

$$= \begin{bmatrix} \dfrac{\frac{cx_1 + dy_1 + \cancel{cx_2} + dy_2}{2} + \frac{cx_1 + dy_1 + cx_2 + dy_2}{2}}{2} \\ -//- \end{bmatrix}$$

$$= \begin{bmatrix} \frac{cx_1 + dy_1 + cx_2 + dy_2}{2} \\ -//- \end{bmatrix}$$

7.2

15 $\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}2\\5\end{bmatrix}$ $\begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}1\\3\end{bmatrix}$

A?

$\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}2\\5\end{bmatrix}$

$\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}1\\3\end{bmatrix}$

$\begin{bmatrix}2\\5\end{bmatrix} \to \begin{bmatrix}1\\0\end{bmatrix}$ $\qquad \begin{bmatrix}A^{-1}\end{bmatrix} = \begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}$

$\begin{bmatrix}1\\3\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix}$

$\begin{bmatrix}2\\6\end{bmatrix} \to \begin{bmatrix}1\\0\end{bmatrix}$ Γραμμικά εξαρτημένα, δεν υπάρχει αντίστροφος

$\begin{bmatrix}1\\3\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix}$

$\xi_1 \xi_2$ Δεν είναι

ξ_1^2

$\xi_1 + (1,1)$ μπορεί

Περσινο θεμα

$V, W \in \mathbb{R}^3$ $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

$T: V \to W$
$T(v_1) = w_2$
$T(v_2) = w_3$
$T(v_3) = w_1$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} w_2 \\ w_3 \\ w_1 \end{bmatrix}$$

προπερσι

$$T\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} 2\xi_1 \\ \xi_2 + \xi_3 \end{pmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} 2\xi_1 \\ \xi_2 + \xi_3 \end{bmatrix}$$

Ποια είναι όμοια

$$\underset{A_1}{\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}} \quad \underset{A_2}{\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}} \quad \underset{A_3}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \quad \underset{A_4}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

$A = XQX^{-1}$ ίδιες ιδιοτιμές

ίχνος $tr(A_1) = 1$ $tr(A_2) = 1$ $tr(A_3) = 2$ $tr(A_4) = 0$

$\lambda_1 = 0$ $\lambda_1 = 0$
$\lambda_2 = 1$ $\lambda_2 = 1$

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$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$A_1 \quad A_2 \quad A_3 \quad A_4$

$tr(A_1)=0 \quad tr(A_2)=0 \quad \cancel{tr(A_3)=1} \quad \cancel{tr(A_4)=2}$

$\lambda_{1,2}=0 \quad \lambda=\pm 1$

κανένα όμοιο

$$\begin{pmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 6 & 0 \\ 1 & 0 & 3 & 0 \end{pmatrix} \to \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & 0 & 6 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = rref(A)$$

$M_1 P_2 L_1 R_1 = \begin{bmatrix} \\ \\ \end{bmatrix}$

Αριστερός Μηδενοχώρος
Αλλιώς A^T και rref

$$range(A) = \left\{ \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\} \qquad range(A^T) = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$Null(A^T) = \begin{bmatrix} 0 \\ -1/2 \\ 1 \end{bmatrix} \quad \text{ή} \quad \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

Επαναληπτικό

$P = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}_{3\times1}$ $q = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}_{3\times1}$

$\underset{3\times1\ 3\times1}{p \cdot q} = X$ | $\underset{3\times1\ 1\times3 = 3\times3}{pq^T} = \checkmark$

$\underset{1\times3\ 3\times1 = 1\times1}{q^T p} = \checkmark$

$A = 4\times4$ B C

$A = (:,1\ :3) = 4\times3$

$A(:,1 \cdot 3)\ B^T$

A 4×2 | AB^T

B 2×5 | $4\times\cancel{2\ 5}\times2 \doteq X$

$$\left.\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_1 - 3x_2 + 4x_3 &= 0 \\ 3x_1 - 5x_2 + 5x_3 &= 0 \end{aligned}\right\} A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 4 \\ 3 & -5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\underset{L_1}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 4 \\ 3 & -5 & 5 \end{bmatrix} \Rightarrow \underset{L_2}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 3 & -5 & 5 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow \underset{U}{\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$L_3 L_2 L_1 = U$

$$x_1 - 2x_2 + x_3 = 0$$
$$x_2 + 2x_3 = 0$$
$$0 = 0$$

Για $x_3 = 1 \Rightarrow r = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 4 & 3 \\ -2 & -6 & 0 \\ -3 & -10 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 6 \\ 0 & 2 & 9 \end{bmatrix} \rightarrow \underbrace{\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}}_{U} \rightarrow A = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Μειωμενη τάξη → Δεν αντιστρέφεται

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow$$

Διαγωνιο κατα πλοκάδες

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}^{-1} = \frac{1}{ad} \begin{bmatrix} d & -b \\ 0 & a \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 \end{bmatrix}$$

$$\begin{array}{cc} A & I \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & -2 & 0 & 1 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & +1 & +2 & +2 & -1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 5 & 4 & -2 \\ 0 & 1 & 0 & -2 & -1 & 1 \\ 0 & 0 & 1 & 2 & 2 & -1 \end{array}\right] \to$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & 5 & -3 \\ 0 & 1 & 0 & -2 & -1 & 1 \\ 0 & 0 & 1 & 2 & 2 & -1 \end{array}\right]$$

$$\begin{array}{cc} I & A^{-1} \end{array}$$

$$A = \begin{bmatrix} -1 & 4 & -1 & +2 \\ -2 & 8 & -2 & 4 \\ -3 & -3 & 4 & -3 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$$

$Ax=b \to A(x+w)=Ax+$

$Aw=0$

$\Uparrow$

μηδενόχωρος $\quad x = x_p + x_{}$

$$\left[\begin{array}{cccc|c} -1 & 4 & -1 & 2 & 4 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & -15 & 7 & -9 & -11 \end{array}\right] \to \left[\begin{array}{cccc|c} -1 & 4 & -1 & 2 & 4 \\ 0 & -15 & 7 & -9 & -11 \\ 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

Δεν έχει λύσ

K

$$\overset{K}{\to} \begin{bmatrix} -1 & 0 & & \\ 0 & -15 & 7 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -13/15 & 6/15 \\ 0 & 1 & -7/15 & 9/15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \left[\begin{array}{c|c} I & F \end{array}\right.$$

rank = 2

$$Null(A) = \left\{ \begin{bmatrix} -13/15 \\ -7/15 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6/15 \\ -9/15 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$range(A) = \left\{ \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ -3 \end{bmatrix} \right\}$$

$$range(A^T) = \left\{ \begin{bmatrix} 1 \\ 0 \\ -13/15 \\ 6/15 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -7/15 \\ 9/15 \end{bmatrix} \right\}$$

Παραγοντοποίηση, rank=2 $\rightarrow 3\times\cancel{2}\ 2\times 4$

$$A = \begin{bmatrix} -1 & 4 \\ -2 & 8 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -13/15 & 6/15 \\ 0 & 1 & -7/15 & 9/15 \end{bmatrix}$$

$x = x_p + x_s \rightarrow Ax=0$ για εναλλαγή των free 1,0

$\hookrightarrow Ax=b$ όλες τις free μεταβλητές=0

$$x = 0 + k \begin{bmatrix} 13/15 \\ 7/15 \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -6/15 \\ -9/15 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{array}{c} \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 2 & 6 & 8 \\ 1 & 1 & 2 \end{bmatrix} \\ \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \end{array}$$ γραμμικά εξαρτημένη, φεύγει

$$q_1 = \frac{a_1}{\|a_1\|} = \frac{1}{\sqrt{7}} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\tilde{q}_2 = a_2 - (q_1^T q_2) q_1 =$$

$$= \begin{bmatrix} 3 \\ 2 \\ 6 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{7}} \Big([1 \ 1 \ 2 \ 1] \begin{bmatrix} 3 \\ 2 \\ 6 \\ 1 \end{bmatrix}$$

$$\tilde{q}_2 = \frac{1}{7} \begin{bmatrix} 3 \\ -4 \\ 6 \\ -11 \end{bmatrix}$$

$$q_2 = \frac{\tilde{q}_2}{\|q_2\|} = \frac{\sqrt{7}}{7\sqrt{26}} \begin{bmatrix} 3 \\ -4 \\ 6 \\ 11 \end{bmatrix}$$

$$Q = \begin{bmatrix} 3/7 & \frac{3\sqrt{7}}{7\sqrt{26}} \\ -4/7 & -4\sqrt{7}/7\sqrt{26} \\ 6/7 & 6\sqrt{7}/7\sqrt{6} \\ -11/7 & 11\sqrt{7}/7\sqrt{26} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -6 & -2 \\ -2 & 7 & 5 \end{bmatrix}$$ Γραμμικά εξαρτημένα

Να προβάλω το $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

α' τρόπος $\left.\begin{array}{l} P = A(A^T A)^{-1} A^T \\ P_b = Pb \end{array}\right\} \rightarrow A = \begin{bmatrix} 2 & 1 \\ 4 & -6 \\ -2 & 7 \end{bmatrix} \rightarrow P = \begin{bmatrix} 2/3 & 1/3 & 1/3 \\ 1/3 & 2/3 & -1/3 \\ 1/3 & -1/3 & 2/3 \end{bmatrix}$

$Pb = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ Βρίσκεται ήδη στο χώρο το b.

Μετά από Gram-Schmidt

$$Q = \begin{bmatrix} 2/\sqrt{24} & 4\sqrt{32} \\ 4\sqrt{24} & 0 \\ -2/\sqrt{24} & 4\sqrt{32} \end{bmatrix} \quad Pb = (b^T q_1)q_1 + (b^T q_2)q_2$$

$\quad q_1 \quad\quad q_2$

Ελάχιστα τετράγωνα

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 26 \end{bmatrix} \quad b \notin \text{range}(A)$$

$$\begin{array}{l} Ax = b \\ A^T A x = A^T b \end{array} \quad \underbrace{\overset{A^T}{\begin{bmatrix} 2 & -2 & 5 \\ -2 & 2 & 3 \end{bmatrix}}}_{2\times 3} \overset{A}{\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 5 & 3 \end{bmatrix}} = \begin{bmatrix} 33 & 7 \\ \cancel{8}7 & 17 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & 5 \\ -2 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 7 \\ 26 \end{bmatrix} = \begin{bmatrix} 114 \\ 94 \end{bmatrix}$$

$$\begin{bmatrix} 33 & 7 \\ 7 & 17 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 114 \\ 94 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ -2 & -2 & -1 \end{bmatrix} \to \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ -2 & -2 & -1-\lambda \end{bmatrix} \to$$

$$(-1)^{1+1}(1-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ -2 & -1-\lambda \end{vmatrix} = (1-\lambda)\left[(1-\lambda)(-1-\lambda)+2\right] =$$

$$= (1-\lambda)(-1-\cancel{\lambda}+\cancel{\lambda}+\lambda^2+2) = (1-\lambda)(\lambda^2+1) =$$

$$= \begin{cases} \lambda = 1 \\ \lambda = \pm i \end{cases}$$

$$P(\lambda) = -\lambda^3+\lambda^2-\lambda+1$$

Θ. Caley hamilto $P(A)=0 \Rightarrow -A^3+A^2-A+I=0 \Rightarrow$

$$\Rightarrow -A^3+A^2-A=-I \Rightarrow A^2-A+I=A^{-1} \Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & -1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$

Διαγωνοποιηση

$$\overset{A}{\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}} \to \begin{bmatrix} 1-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{bmatrix} \to (-1)^2(1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} + \begin{vmatrix} 1 & \\ 1 & \end{vmatrix}$$

$$= (1-\lambda)\left[(2-\lambda)(-1-\lambda)-1\right] -1-\lambda+2 =$$

$$= (1-\lambda)(-2-2\lambda+\lambda+\lambda^2-1) -\lambda+1 =$$

$$(1-\lambda)(\lambda^2-\lambda-3)-\lambda+1 = \lambda^2-\lambda-3-\lambda^3+\lambda^2+3\lambda-\lambda+1 =$$

$$= -\lambda^3+2\lambda^2+\lambda-2 = (1-\lambda)(\lambda+1)(\lambda-2)$$

$$\lambda_1 = -1 \rightarrow \begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix} x = 0 \rightarrow v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 1 \rightarrow \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} x = 0 \rightarrow v_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 2 \longrightarrow v_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$A = \underbrace{\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}}^{X} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} X^{-1}$$

$$A = \begin{bmatrix} -3 & 6 & 6 \\ -3 & 6 & 6 \\ -2 & 4 & 4 \end{bmatrix}$$ α) βρες ιδιάζουσες

α) $\cancel{A}$ $\begin{bmatrix} -3 & 6 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$A = \underbrace{\begin{bmatrix} -3 \\ -3 \\ -2 \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} 1 & -2 & -2 \end{bmatrix}}_{b^T} \qquad A = a b^T$$

$$SVD \rightarrow \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \sigma_3 \end{bmatrix} \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix}$$

$$A = u\sigma_1 v_1^T + \cancel{u_2 \sigma_2 v_2^T} + \cancel{u_3 \sigma_3 v_3^T} \quad (\text{rank}=1)$$

$$\text{SVD} \to A = \sigma_1 u_1 v_1^T$$

$$\sigma_1 = \|a\|_2 \|b\|_2 = 3\sqrt{22}$$

Ορίζουσα 0
τάξης 1

β) $B = u_1 v_1^T \to A = \sigma_1 u_1 v_1^T \Rightarrow A = \sigma_1 B \to B = \frac{1}{\sigma_1} A$

Ψευδοαντίστροφος

$$A^+ = (U\Sigma V^T)^T = V\Sigma^+ U^T$$

$$A^+ = (\sigma_1 u_1 v_1^T)^T = \frac{1}{\sigma_1} \underbrace{v_1 u_1^T}_{B}$$

$$T\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 + 1 \end{pmatrix}$$

$v, w \in R^3 \quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

$$T(cv + dw) = T\left(c\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + d\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right) =$$

$T(0) = 0$ X

$$T\left(\begin{bmatrix} cv_1 + dw_1 \\ cv_2 + dw_2 \\ cv_3 + dw_3 \end{bmatrix}\right) = \begin{bmatrix} cv_1 + dw_1 \\ cv_2 + dw_2 \\ cv_3 + dw_3 + 1 \end{bmatrix} = c\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + d\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} +$$